A Proof Sketch for Lemma 1

In this section, we prove the omitted proof sketch of Lemma 1:

Proof sketch. The continuity is implied from the discussion thus far, where the original continuity of the boundary curve has been preserved. As to strict monotonicity, we proceed in two steps. First, the θ parameter must vary while traversing a VGM. This is simple as $\sim_{\mathrm{p}}$ identifies all gaps originating from the same polygonal edge with the same direction. After the quotient operation, the remaining gap rays all have their origins on a cusp or a point with curvature. Thus, the θ parameter of neighboring rays is never constant. Second, the change of θ is monotonic. For a (p, θ) that is not an endpoint of M, the (p, θ) bisect its neighborhood into two subintervals: it can only violate monotonicity if the both neighborhoods have angle parameter less than θ, or both neighborhoods have angle parameter greater than θ. Suppose either of these cases occur, and (p, θ) is a differentiable point. Then it meets the definition of a differentiable inflection point, and hence Assumption 1 implies we can find, on either side, a neighborhood that is entirely concave or convex. But the inflection requires that one be concave and other be convex. Then the concave neighborhood violates visibility, so must be outside of M as $\mathrm{M} \subseteq \mathcal{M}_v(\mathcal{E})$. This, thus, contradicts the interiority of (p, θ). The situation when (p, θ) is a non-differentiable point is similar: it meets the definition of a non-differentiable inflection point, and the argument is analogous, albeit with additional care to consider angles that are extremal. For monotonicity to be violated by an endpoint, it would need to break continuity. [End of sketch]

B Proof Sketch for Lemma 2

In this section, we prove the omitted proof sketch of Lemma 2:

Proof sketch. We first characterize VGM endpoints. A point (p, θ) is an endpoint of M only if both neighboring gaps have angle parameters either all less than θ or all greater than θ. If instead the neighbors had angles on opposite sides of θ, then similar to the argument in the previous proof, such (p, θ) can not be the endpoint of a VGM, as the neighboring gap rays are either all visible or all non-visible on both sides of (p, θ).

We now establish the directionality. Let (p, θ) be an endpoint of M with $p = \varphi_c(t)$ on boundary curve c. If p is differentiable, the neighborhood of p on $\partial\mathcal{E}$ is strictly concave on one side and strictly convex on the other. Whether $(p, \theta) \in \mathrm{M}$ depends on which side is convex and whether M is an L-VGM or R-VGM. Suppose M is an R-VGM. If the convex side corresponds to $\tau > t$, then $(p, \theta) \in \mathrm{M}$, forming a closed endpoint; the angle decreases as a gap approaches (p, θ), meaning the endpoint is reached by moving in the revealing direction. Symmetrically, if M is an L-VGM and the convex side has $\tau < t$, then (p, θ) is also a closed endpoint of M reached in the revealing direction. The other endpoint has the convex neighborhood on the opposite side, forming an open endpoint approached in the concealing direction.

If M ends on a non-differentiable point, a similar argument applies by taking into consideration the gaps in the neighborhood of (p, θ) that emanate from p.

[End of sketch]

C The Proof Sketch of Lemma 3

Proof sketch. Relation $\prec$ ensures consecutive events involve the same VG-state traversing intervals that are adjacent. Each revealing event advances the VG-state one interval in the revealing direction; each concealing event reverses this. With equal counts, the VG-state returns to its original interval whence it departed in event e_1. As e_1 is a GAPSPLIT and e_{2m} is a GAPMERGE, they are dual events. [End of sketch]

D The Proof of Theorem 1

In this section, we prove the omitted proof for Theorem 1:

Proof. We proceed by induction on the trajectory length.

Base case: At I_1, the algorithm creates a child of the root for each observed gap. Since I_1 is a star configuration, no interval is a strict revealing descendant of another, so in a simply-connected environment, the interval nodes in I_1 are not connected by concealing edges in $\mathbb{E}(G_v(\mathcal{E}))$. This matches the GNT structure: these nodes are not connected to each other. Setting f to map each node to its corresponding interval satisfies both properties.

Inductive step: Assume the theorem holds at configuration I_i. We show it holds at I_{i+1} after edge e_i.

Case $e_i = (\iota, \top)$ (GAPDISAPPEAR): The algorithm removes the leaf node g with $f(g) = \iota$. Removing g preserves both properties.

Case $e_i = (\top, \iota)$ (GAPAPPEAR): A new node g is added to the root with $f(g) = \iota$. Since I_{i+1} is a star configuration, ι is not a strict revealing descendant of any other interval in I_{i+1}, so ι is not connected to any existing $f(g')$ in $\mathbb{V}(G_v(\mathcal{E}))$. Thus g is correctly placed as a child of the root, preserving property (1). Property (2) holds since ι has no revealing descendants.

Case $e_i = (\iota, (\iota', \iota''))$ (GAPSPLIT): Let g be the node with $f(g) = \iota$. If g is a leaf, it is replaced by nodes g' and g'' with $f(g') = \iota'$ and $f(g'') = \iota''$; property (1) holds as ι' and ι'' are not connected by concealing edges in $\mathbb{E}(G_v(\mathcal{E}))$. Since ι has no visited revealing descendants, neither do ι' and ι'', and property (2) holds. If g has children, they were added at a previous GAPMERGE $((\iota', \iota''), \iota)$. By Lemma 3, this GAPSPLIT is the dual of that GAPMERGE. The algorithm removes g and re-associates its children with g' and g'' by matching proximal/distal edge types. Since the edge types are fixed, the re-association is correct: children previously connected via edges involving ι' attach to g', and similarly for ι''. The function f remains valid, and property (1) is preserved. Moreover, the visited revealing descendants of ι' and ι'' are contained in those of ι, which already exist in G, hence property (2) is preserved.

Case $e_i = ((\iota', \iota''), \iota)$ (GAPMERGE): Nodes g' and g'' with $f(g') = \iota'$ and $f(g'') = \iota''$ become children of a new node g with $f(g) = \iota$, connected by proximal and distal edges matching the event. Since ι' and ι'' are strict revealing descendants of ι, and they and their strict revealing descendants are exactly the strict revealing descendants of ι, and property (2) holds. Property (1) holds by construction. □