

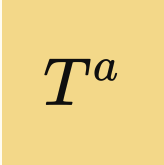


# Discrete POMDP

  
 $b_0^T$   
  
(initial belief distribution)

$$\left\{ T^a : a \in \mathcal{A} \right\}$$

  $T^a$  row-stochastic.

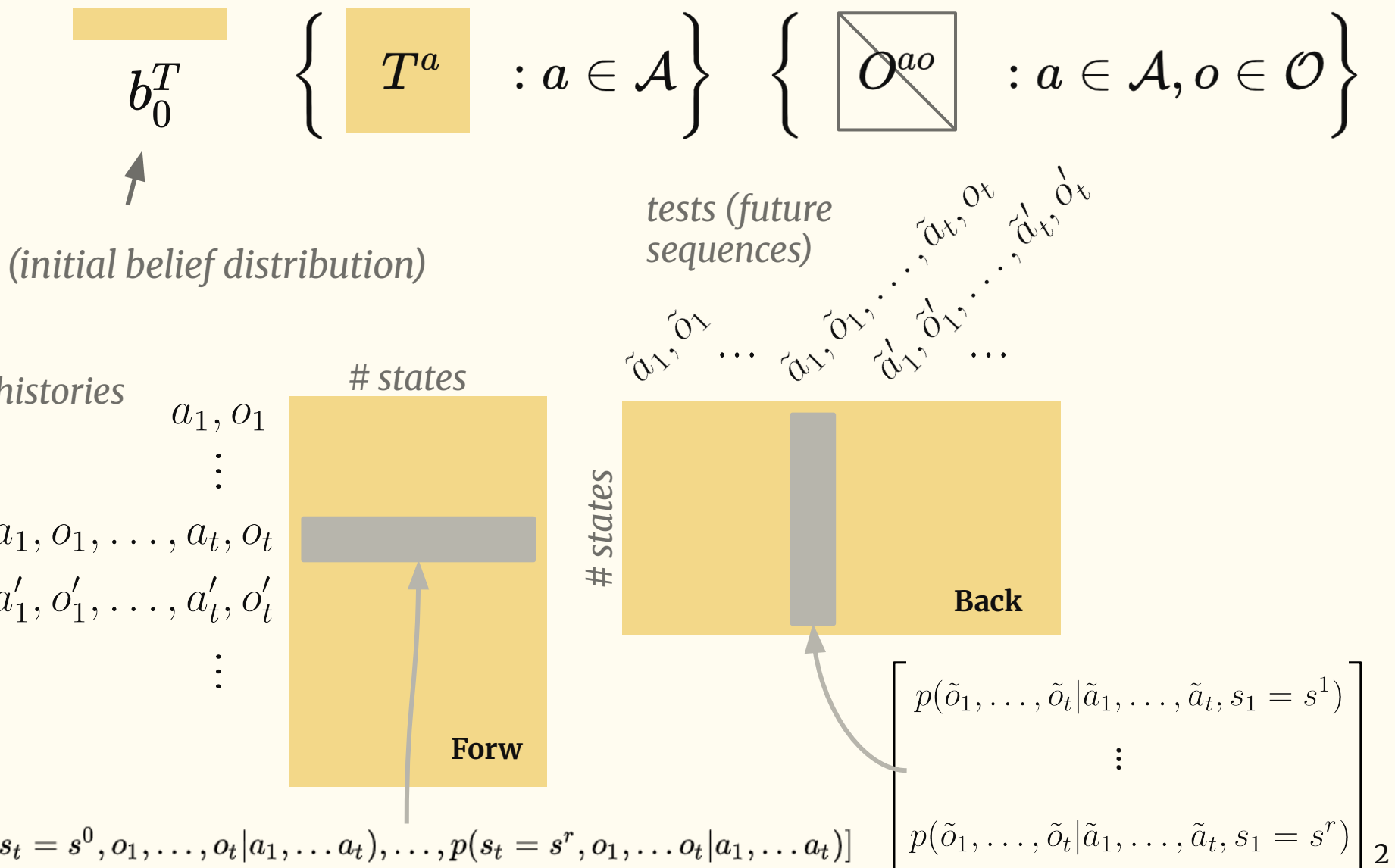
$$\left\{ O^{ao} : a \in \mathcal{A}, o \in \mathcal{O} \right\}$$

  $O^{ao}$  diagonal.

$$O_{ii}^{ao} = p(o|a, s = i)$$

# Forward and Backward Matrices

From the discrete-state POMDP



# Forward and Backward Matrices

From the discrete-state POMDP

$$b_0^T \quad \left\{ T^a : a \in \mathcal{A} \right\} \quad \left\{ O^{ao} : a \in \mathcal{A}, o \in \mathcal{O} \right\}$$

tests (future sequences)

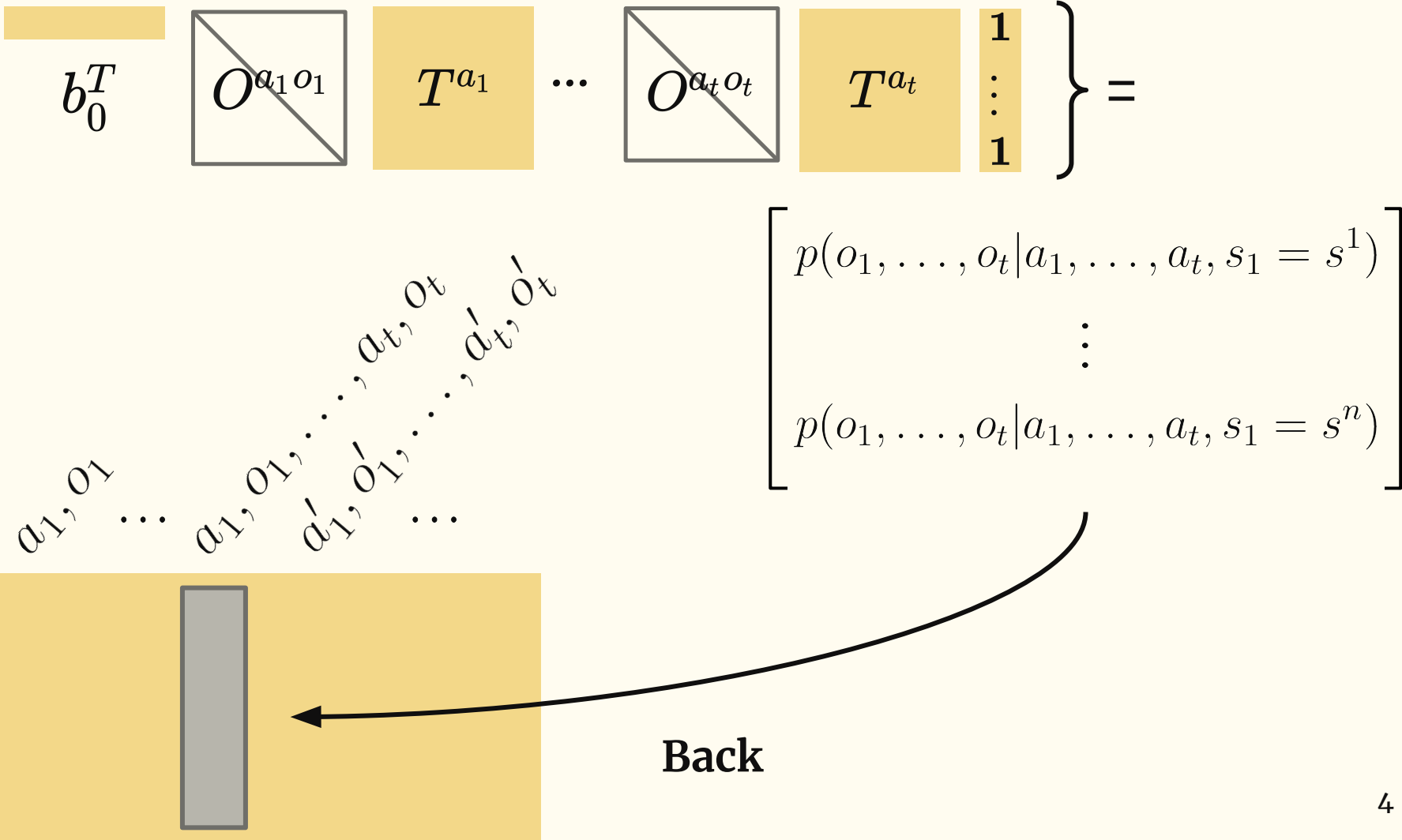
$\tilde{a}_1, \tilde{o}_1 \dots \tilde{a}_1, \tilde{o}_1, \dots, \tilde{a}_t, \tilde{o}_t$   
 $\tilde{a}'_1, \tilde{o}'_1 \dots \tilde{a}'_1, \tilde{o}'_1, \dots, \tilde{a}'_t, \tilde{o}'_t$



$$\begin{bmatrix} p(\tilde{o}_1, \dots, \tilde{o}_t | \tilde{a}_1, \dots, \tilde{a}_t, s_1 = s^1) \\ \vdots \\ p(\tilde{o}_1, \dots, \tilde{o}_t | \tilde{a}_1, \dots, \tilde{a}_t, s_1 = s^r) \end{bmatrix}_3$$

# The backward matrix (Back)

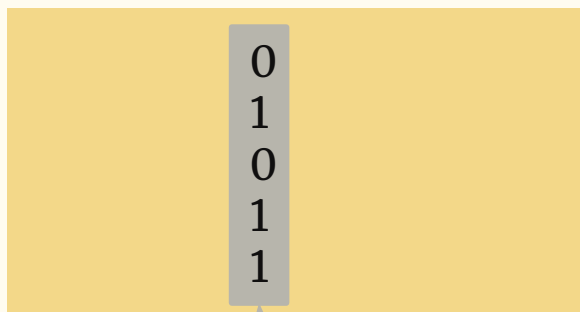
If we did not multiply by the initial belief...



# Backward Matrix is Deterministic

tests (future sequences) –  
all sequences  
enumerated up to some  
length  $K$

$\tilde{a}_1, \tilde{o}_1 \dots \tilde{a}_1, \tilde{o}_1, \dots, \tilde{a}_t, o_t$   
 $\tilde{a}'_1, \tilde{o}'_1, \dots, \tilde{a}'_t, \tilde{o}'_t$



$$\begin{bmatrix} p(\tilde{o}_1, \dots, \tilde{o}_t | \tilde{a}_1, \dots, \tilde{a}_t, s_1 = s^1) \\ \vdots \\ p(\tilde{o}_1, \dots, \tilde{o}_t | \tilde{a}_1, \dots, \tilde{a}_t, s_1 = s^r) \end{bmatrix}$$

You can use the backward matrix to find ‘minimal’ POMDPs in # of states.

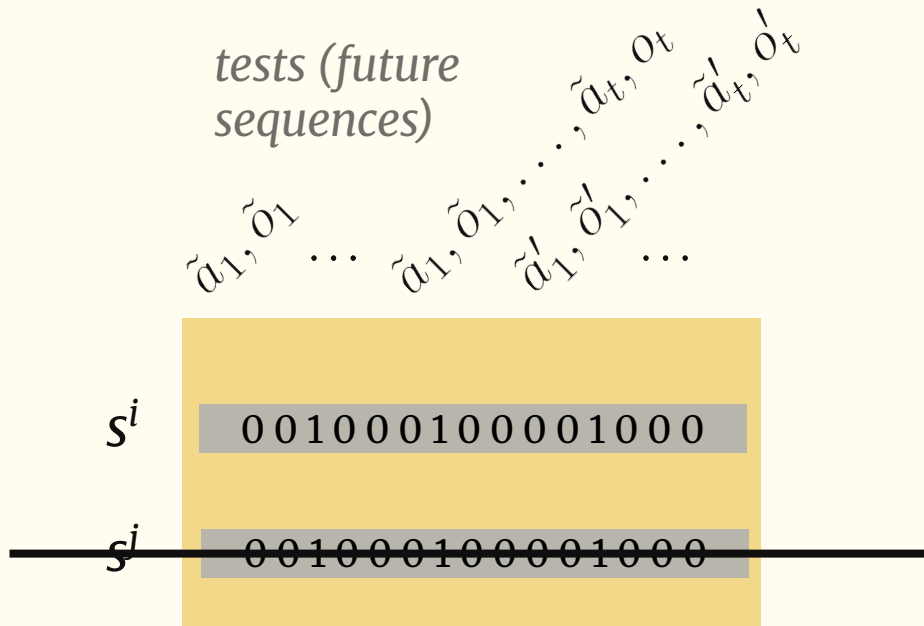
tests (future sequences)  
 $\tilde{a}_1, \tilde{o}_1 \dots \tilde{a}_1, \tilde{o}_1, \dots, \tilde{a}_t, o_t$   
 $\tilde{a}'_1, \tilde{o}'_1 \dots \tilde{a}'_1, \tilde{o}'_1, \dots, \tilde{a}'_t, o'_t$

$s^i$	001000100001000
$s^j$	001000100001000



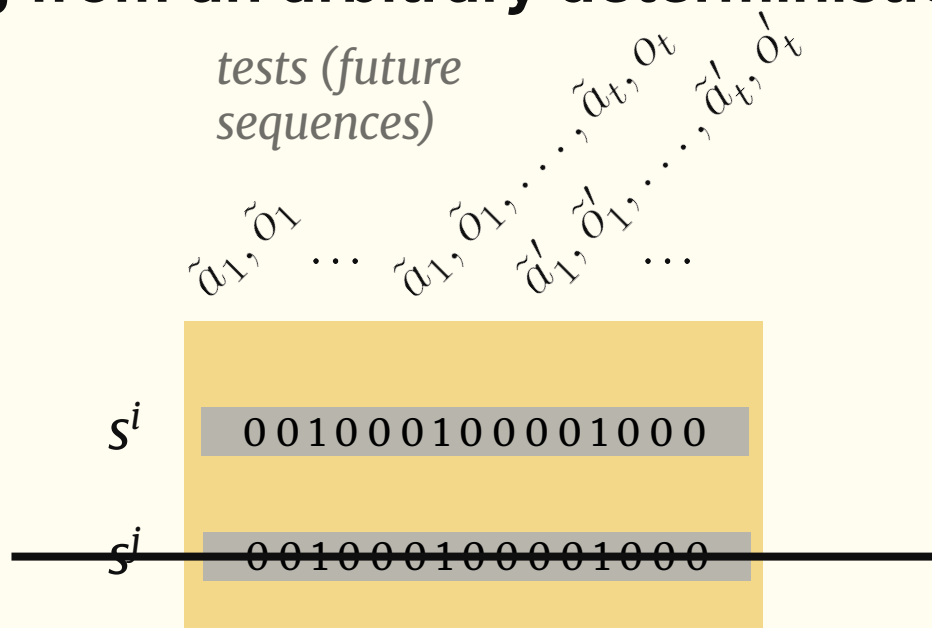
Just delete duplicate rows.

You can use the backward matrix to find ‘minimal’ POMDPs in # of states.



Just delete duplicate rows.

**Question: is this matrix always full-rank (= # of rows), starting from an arbitrary deterministic POMDP?**



You can assume vector spaces not just over the reals.  
 $F_2$  is also fine.